Long-Term Prediction of GPS Accuracy: Understanding the Fundamentals

Ted Driver, *Analytical Graphics Incorporated*

BIOGRAPHY

Ted Driver is the Senior Navigation Engineer at Analytical Graphics Inc. Ted has worked on AGI's navigation capabilities for over three years, having previously been the technical lead for Navigation Tool Kit development at Overlook Systems Technologies. He has led the engineering team in developing the navigation algorithms and data stream definitions and is currently working on statistical prediction models for GPS accuracy. He was previously the senior GPS Operations Center analyst within the 2nd Space Operations Squadron at Schriever Air Force Base. He has worked in the GPS field for 10 years, previously designing the environment and navigation models for the GPS High Fidelity Simulator currently in use at Schriever Air Force Base. Mr. Driver received his Bachelors Degree in Physics from the University of California at San Diego and his Masters Degree in Physics from the University of Colorado. He is a former President, Vice President and Secretary of the Rocky Mountain Section of the Institute of Navigation.

ABSTRACT

This paper will explore the obstacles inherent in predicting GNSS-based navigation accuracy, discussing each obstacle in turn and attempting to bound navigation errors as a function of time. Depending on your definition of accuracy – this topic can be relatively easy or terribly difficult. For this analysis, we will focus on major contributors to navigation positioning errors, such as Dilution of Precision (DOP), and User Range Error (URE) behaviors.

This paper is born out of the nascent need by organizations to plan missions based on future navigation accuracy – a need that is not easily met currently. I will look at the behaviors of each error contributor to the accuracy prediction problem, and attempt to mathematically or statistically bound the error produced by each. Complexities such as the non-Gaussian behavior of the errors must be addressed as well. The predictions will take on two forms: extrapolated instantaneous errors and statistical errors mapped to specific confidence levels. We'll also show how these bounds can be converted to other accuracy parameters using standard methods.

Some of the elements that make up the total navigation accuracy picture can be bounded fairly easily, given a few modeling parameters. Others are not so tame and will exhibit complex behaviors. I anticipate the results will show large variances in the different predicted error bounds and that this in turn will drive future work in the area. I anticipate writing follow on papers discussing the specific details of the different error contributors in the near future and encourage others to do so as well.

Navigation error prediction is becoming more prevalent and as users get more sophisticated, dilution of precision predictions are no longer sufficient for their needs. Providing a framework to work from, this paper will allow users of position error predictions to better understand their specific problem. This work will also lead to further research in accuracy prediction, forming a foundation from which to grow.

INTRODUCTION

Two subjective terms are used in the title of this paper; Long-Term and Accuracy. Differing individuals or groups will have different definitions for both. I do not have a strict definition myself, so I'll let my examination of the data provide some answers.

Radio-navigation position accuracy consists of several error sources, with error budgets well defined in many texts. [1][2][3] Finding a complete theory of navigation error prediction will necessarily require that all error sources be predictable – to one degree or another. In theory, once the different error sources can be predicted, the problem is complete, all that would remain is to combine these error predictions in some meaningful fashion to determine the entire navigation error problem at some future time. References on the propagation of errors are good places to begin that study [10]. Of course when theory meets practice, problems inevitably arise. Let's start with a list of the common error sources and see where we start to have trouble. Once we find the trouble, let's see if we can find ways around the issues or somehow bound the problem.

NAVIGATION ERROR SOURCES

This error source list is not meant to be all-inclusive, but includes those with the largest effects

- Dilution of Precision
- Signal-In-Space Range Error
  - Ephemeris Error
  - Clock Error
- Atmospheric Error
  - Ionospheric Error
  - Tropospheric Error
- User Equipment Error
  - Multipath
  - Receiver noise

For this paper, I will not look at the atmospheric behaviors. This is a topic in and of itself [9]. Let's look at each of the remaining errors in turn.

Dilution of Precision

Many references provide details regarding what dilution of precision (DOP) is and how it arises [1] [2] [3]. For this paper, I'll assume the reader is familiar with DOP and how it affects navigation accuracy. For the most part, DOP is predictable, since it arises from constellation geometry alone. All that's required to predict DOP is to be able to predict the GPS satellite orbital positions and even then no great accuracy is required.

Note that it is assumed here that there are no obstructions to your obtaining the predicted DOP value. Physical obstructions as well as radio frequency obstructions can limit your receiver's ability to track all satellites that could otherwise be seen. This paper's analysis will assume that none of those obstructions are in place and all satellites predicted to be tracked are actually tracked.

Differences in the actual positions of the satellites on the order of tens of kilometers lead to DOP differences only in the 3rd and 4th decimal place. This is provided of course that the almanac propagation is close to the Time of Almanac (TOA) [4] when compared to the precise ephemeris.

Here again is a subjective term – close. Let's look a little closer at DOP values predicted with an almanac propagated into the future some number of weeks against the actual DOP value for that time calculated from the National Geospatial-Intelligence Agency (NGA) precise ephemeris. This analysis will show us how far we can use a single almanac, before it starts affecting our accuracy. Figure 6.3 in [2] shows that position accuracy is linearly dependent on dilution of precision. The dilution of precision error then has a direct affect on position errors. Figure 1 shows the position DOP residual error plotted against time for 22 weeks of prediction. Figure 2 is a closer look at the first 8 weeks.

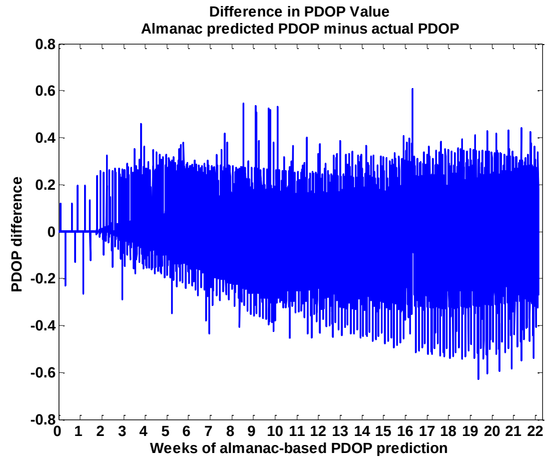

Figure 1 - PDOP Residuals 22 Weeks

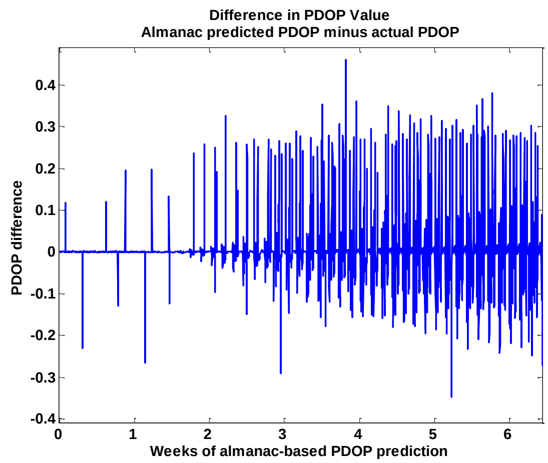

Figure 2 - PDOP Residuals 8 weeks

Upon inspection, the first two weeks of DOP values have a very small residual error – roughly 0.003 peak-to-peak (excluding the few spikes present in this range). As time progresses however the spikes gain control and eventually dominate the residual plot. However, even at 22 weeks out, the absolute DOP prediction error is still under 0.6. Since the DOP residual error is pretty low, even 22 weeks out, let's see how well the predicted DOP values correlate with the actual DOP values.

Figure 3 shows a cross-correlation plot of the predicted, almanac DOP values to the actual DOP values. The interesting portions of this plot are the spikes that occur at daily intervals. One would expect the DOP values to be correlated at the same time each day (more precisely at the 23:59:56 mark) – for at least the first few weeks, and this is what we see. Note though that the correlation linearly decreases until at 22 weeks, where there is little correlation between the DOP values calculated by the almanac prediction and the actual DOP. The magnitude of the DOP value may only change by 0.6 peak-to-peak, but the lack or correlation tells us that the actual DOP data may have spikes where the predicted data showed none.

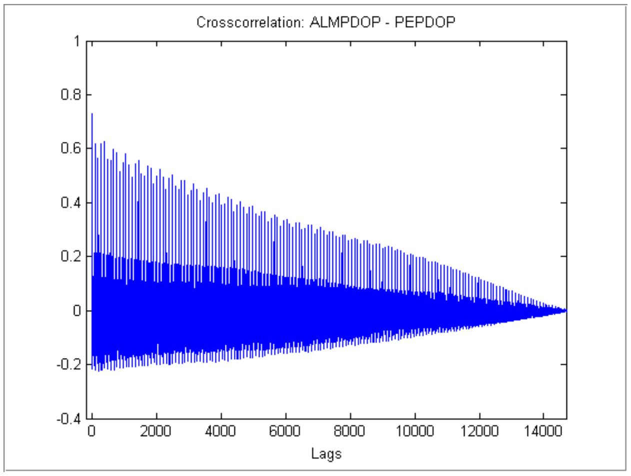

Figure 3 – Predicted PDOP, Actual PDOP Cross Correlation

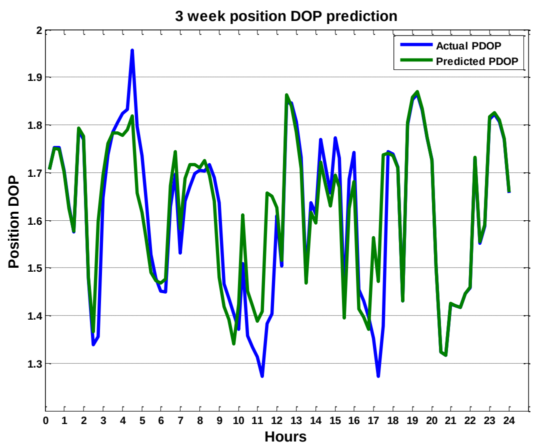

Figure 4 - 3 week DOP prediction example

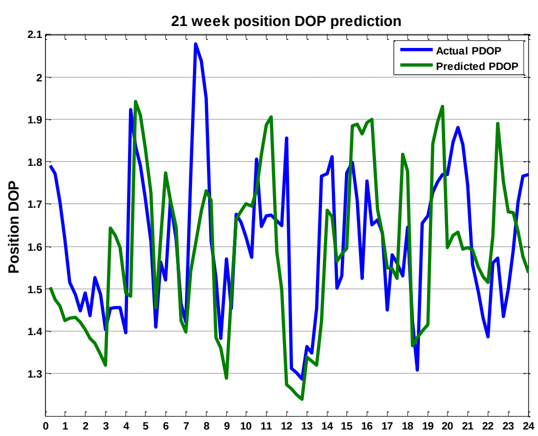

Figure 5 - 21 week DOP prediction example

Figures 4 and 5 show examples of the predicted DOP compared to the actual DOP. Prior to the two week boundary, the predicted DOP is virtually indistinguishable from the actual DOP – the graphs overlay each other.

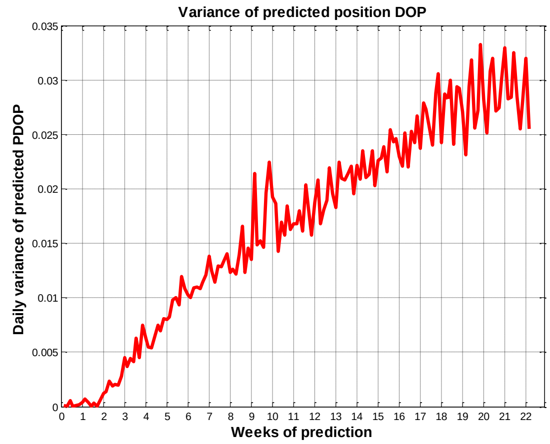

Figure 6 - Variance of Predicted Position DOP

Figure 6 shows the variance of the predicted position DOP by week of prediction. As the prediction time increases, the variance of the predicted DOP increases linearly. Another clear marker here is that the variance is practically zero for the first two weeks. The wise reader will note here that the almanac can be safely used for two weeks or so before the almanac orbit predictions start to degrade user accuracy by providing increasingly incorrect DOP values. So, for DOP analysis, we can define *Long-Term* as two weeks.

As we'll see further on in the paper, DOP is *the* defining criteria for predicting GPS accuracy. While DOP cannot give you a precise navigation error in and of itself, the extent to which we can predict DOP directly ties to the extent we can predict navigation accuracy.

Signal-In-Space Range Error

The SIS ranging error consists of two primary pieces, ephemeris error and clock error. The ephemeris errors are errors between the actual GPS satellite position and the satellite position broadcast to receivers. The clock error is similar – it's the difference between the actual clock phase and the clock phase that's calculated from parameters sent to the receiver. These errors are typically a few meters but can be much more, especially in the case of the clock. Ephemeris errors result from unmodeled perturbations on the satellite and are reduced to almost zero when a new navigation upload is made to the satellite. At this point, the age of data (AOD) is zero and the broadcast ephemeris is at its most accurate. As time progresses throughout the day, imperfections in the ephemeris prediction slowly appear, leading to larger ephemeris errors.

The clock errors act similarly. The clock errors arise from quantum mechanical fluctuations in the atomic clock itself, leading the clock phase to exhibit a random walk behavior. This effect is difficult to predict and over days, and over weeks and months would be impossible to determine.

In this analysis, the job of having to predict the ephemeris and clock errors is made simpler by the fact that these errors are *clamped*. The 2nd Space Operations Squadron (2SOPs) watches both the ephemeris and clock residuals in near real time and ensures that they stay below certain thresholds by uploading new navigation data predictions to the satellites. Typical satellites are uploaded once per day; some more often (usually the older satellites), some less often. This clamping effect on the SIS errors makes predicting long term behavior easier, in that we do not need to be able to predict random clock behaviors for weeks at a time. Under nominal conditions, we can assume a worst case set of errors for the ephemeris and clocks based on an analysis of the long-term trends of the data. To that end, I have analyzed over 800 days of ephemeris and clock errors from 2SOPs and looked at the absolute maximum ephemeris and clock errors for each satellite. See Figures, 7, 8 and 9. Indeed, one can see that the errors do not run off past certain boundaries. Of course, each PRN exhibits a different bound. If 2SOPs did not upload the satellites on a regular basis, these plots would look markedly different. It should be noted that data for all satellite outages was removed for this analysis.

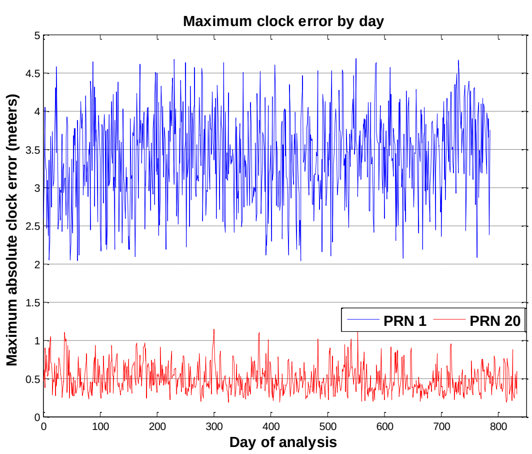

Figure 7 – Sample of Maximum Clock Error by Day

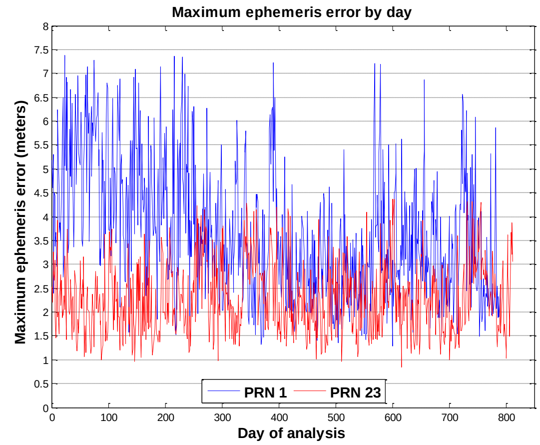

Figure 8 – Sample of Maximum Ephemeris Error by Day

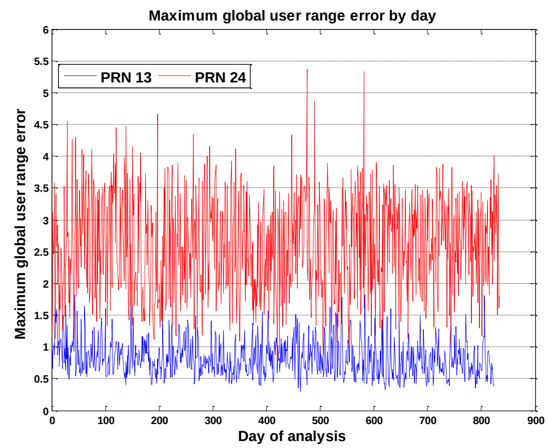

Figure 9 – Sample of Maximum Global User Range Error by Day

The averaged maximum values by satellite [5] are shown in Figures 10, 11 and 12. These plots show the mean value one could use in a prediction scheme for long-term navigation errors by satellite. For example, when predicting the SISURE component of the navigation error using PRN 19, one needn't allow the predicted error to raise much above 0.75 meters. The data here has shown that the maximum global URE error for PRN 19 has consistently been in this range.

This data is helpful in the long-term prediction regime; prediction times longer than a day. In fact, since our certainty of the general clock phase state decreases as time increases (assuming no clamping), this maximum error information becomes more valuable as time increases. In the next section I'll discuss extrapolated predictions in the range from 1 minute to 12 hours to see how to better describe navigation errors in this regime.

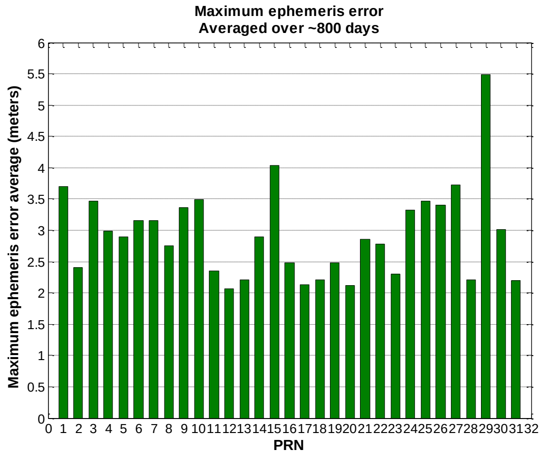

Figure 10 – Averaged Maximum Ephemeris Error

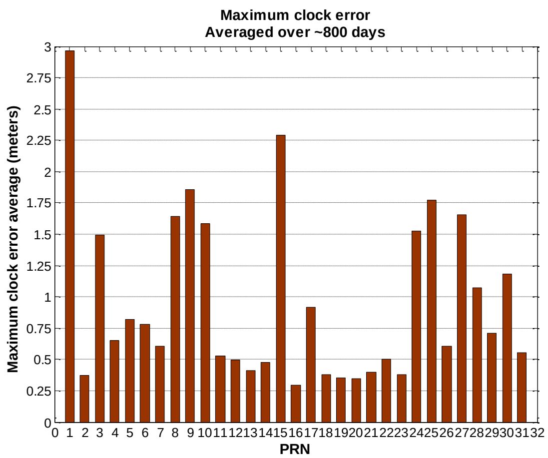

Figure 11 – Averaged Maximum Clock Error

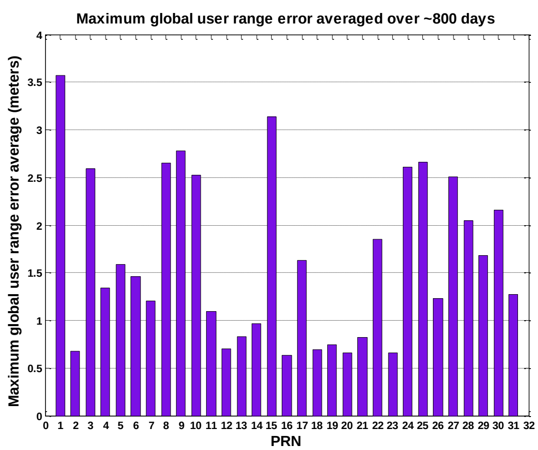

Figure 12 - Averaged Maximum Global User Range Error

User Equipment Errors

The User Equipment Errors (UEEs) are the least predictable of the error sources in our problem. Receiver noise is generated by the receiver tracking loops as they track code, phase and frequency in a variety of dynamic, signal-rich environments. Multipath error results from the receiver receiving multiple signals from the same spacecraft – along different reflected and refracted paths. The receiver can use only those signals within a specific time of reception rendering the other signals as noise the receiver must endure.

Receiver noise error is dependent on many factors that change as a function of time; temperature, g-loading, antenna positioning, etc. Multipath error is dependent upon knowing the exact position of reflective surfaces surrounding the receiver antenna and the orientation of the antenna itself. Determining all of these parameters to produce a viable navigation error prediction is a significant task and presents many challenges – though good work is being done in this area [11]. One way we may attack this problem currently is to take a lesson from the physicists of the 19^{th} century. By then, the laws of classical mechanics could aptly describe all the motions of the particles of a gas in a box, but the sheer number of gas particles precluded the scientists of the time from conducting such a calculation. In their case, they resorted to using the methods of the Statistical Mechanics branch of the discipline. With this, they had to be content with understanding the statistical behaviors of the gas, rather than the explicit motion of each gas particle. In our case, we can do something similar. Instead of trying to understand each multipath reflection, we can create ensemble behaviors for different categories of environments and use root-mean-square (RMS) values derived from these environment types as an additional navigation error. This approach could lead to more efficient calculation schemes than current ray tracing algorithms – though this method will not be able to provide us with instantaneous errors, only statistical behaviors. The field of noise and multipath prediction is nascent and difficult. For this paper, I'll suffice to leave it at being able to add an error value for either noise or multipath or both to the navigation error prediction. Determining ensemble behaviors for different classes of environments is beyond the scope of this paper, but a good topic for a follow-on paper [9].

PREDICTION METHODS

We've looked at the data necessary to perform the predictions, now we need to understand how to predict navigation errors and within which time regimes the answers are viable. It's important to understand how best to predict errors at different times in the future. One method may lead to more accurate results in one regime, and another method may work better at a different time. To better understand the time regimes, let's look at the types of data available to us from which can make predictions. The analysis of this data should naturally point us to times when it should be used.

Prediction Input Data
There are three types of data most likely to be available for predictions:

1) Information about the navigation errors from the previous time step, either from some differential network, or by some other means
2) Statistical information on the errors from previous days
3) The maximum error information from the previous section of this paper

These three choices provide different ways to predict the navigation error at a given time. If all types of data are available, it may be possible to switch prediction techniques based on the time of prediction. Let's look at each of these data types in turn.

Performance Assessment File Data (option 1)
I'm using the term performance assessment file (PAF) because the GPS Operations Center (GPSOC) produces PAF files containing ephemeris and clock errors for each satellite in near-real time. Using this type of standardized data, one can produce the instantaneous navigation errors for a given time. The GPSOC performs these calculations daily. To understand how this data can be used for predictions, we must come up with some type of extrapolation scheme for the ephemeris and clock errors contained in the file. Fortunately, the ephemeris and clock error rates are also included in the PAF file. Using the following simple PAF extrapolation algorithm, I'll propagate the ephemeris and clock error states into the future, and then calculate the user range error and navigation accuracy based on these propagated errors. The notation follows [6].

$$\hat{\vec{E}}_N(h) = \vec{E}_N(0) + \frac{d\vec{E}_N(0)}{dt} \cdot h \cdot \delta t$$

Here, $\hat{\vec{E}}_N(h)$ is the predicted ephemeris error, from time N predicted h steps into the future and δt is the time step of the data. A similar equation holds for the clock data.

Once we have the predicted ephemeris errors and clock errors, we need to create user range errors. The user range errors are created by dotting the predicted ephemeris error vector into the line of sight vector from the receiver to the GPS satellite. This dotted quantity then has the predicted clock error subtracted from it.

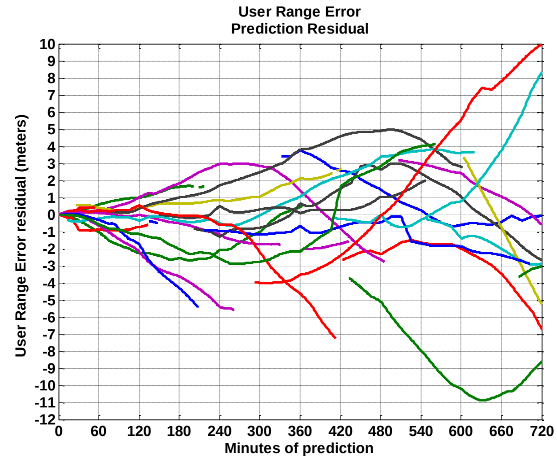

Figure 13 – User Range Error Residuals

Figure 13 shows how the user range error prediction residuals behave as the prediction time h increases from 0 to 12 hours. Within the first hour, the URE residuals do not vary by more than ±1 meter. These errors grow as the prediction time increases. To see how these predicted UREs affect the predicted navigation performance, see Figures 14 and 15.

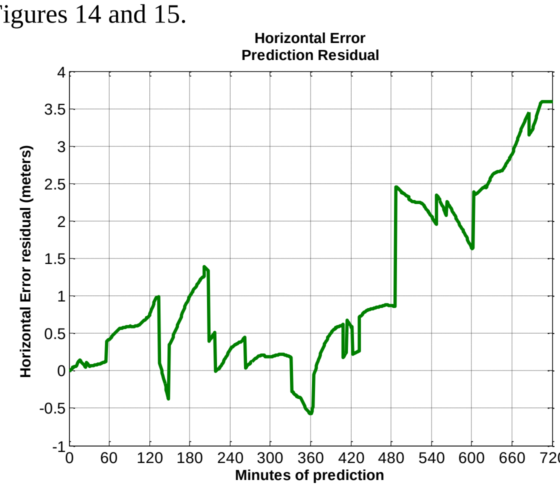

Figure 14 - Horizontal Error Residual

For roughly the first 6 hours (360 minutes), the navigation error residuals are within a few meters. This single day-single site analysis shows at a rudimentary level, how the PAF extrapolation method can be used for navigation accuracy prediction. However, a more detailed analysis is needed here to understand how regional effects and daily effects average out over the long term. It does appear though that even with a detailed analysis we are not going to get much better than 6-12 hours of predictability using this method and expect to maintain an accuracy level expected by the majority of GPS users.

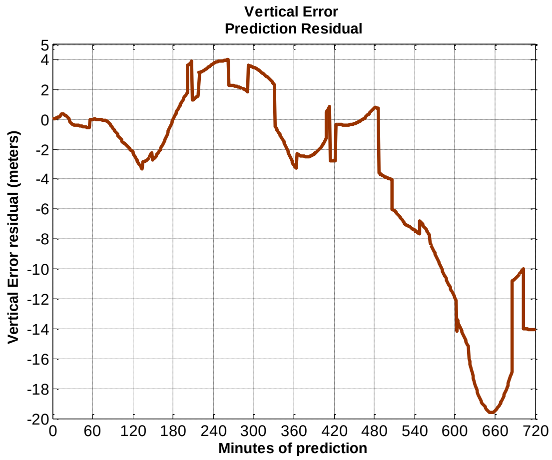

Figure 15 - Vertical Error Residual

Prediction Support File Data (option 2)
The GPSOC also produces statistical data for each GPS satellite's performance. In particular the Prediction Support File (PSF) contains the 1-sigma errors for the radial, along-track and cross-track components, as well as for the global user range error and clock error over the last seven days. The global user range error is defined as:

$$URE_G = \sqrt{\Delta Clock^2 + 0.96 \cdot \Delta R^2 + 0.02 \cdot \left(\Delta A^2 + \Delta C^2\right) - 1.96 \cdot \Delta R \cdot \Delta Clock}$$

Equation 1 - Global URE

Here, URE_G is the 7-day 1-sigma global user range error, $\Delta Clock$ is the 7-day 1-sigma clock error, ΔR is the 7-day 1-sigma radial error, ΔA is the 7-day 1-sigma along-track error and ΔC is the 7-day 1-sigma cross-track error. The global user range error equation derives from integrating the user range error over the entire face of the Earth.

This statistical data can be used to make statistical predictions of navigation accuracy. We will not be able to get instantaneous errors as we did with extrapolated PAF data, but we can predict navigation errors with a specified confidence level.
Using the constant 1-sigma value for the URE_G for each satellite, we can calculate the 1-sigma value for our navigation error into the future. If we predict the vertical error or the time error, our prediction will have a confidence level of 68.27% since both time and vertical errors are 1-dimensional quantities. If however, we predict horizontal error, a 2-dimensional quantity; our 1-sigma prediction will have a confidence level of 39.35%. The predicted position error, a 3-dimensional value, will have a 1-sigma confidence level of 19.9%. To be able to measure the effectiveness of these predictions, we need to convert these 1-sigma error predictions to some standard confidence interval. Typically, the confidence levels used are 50% and 95%. Standard conversion multipliers exist [7] [12], however these standard multipliers are derived from normal, Gaussian processes. Unfortunately, GPS errors are not well represented by Gaussian statistics in the long term. Referring to Figure 16, I've created a histogram of roughly 20,000 position errors, and then plotted several best-fit distributions against the position error data. It's obvious that the Normal distribution is not well-suited; however, the usually quoted Rayleigh distribution is not the best fit either. The Weibull fit is roughly equivalent to the Rayleigh fit (the Weibull distribution is a generalization of the Rayleigh distribution [8]) but the Gamma fit seems to be the best.

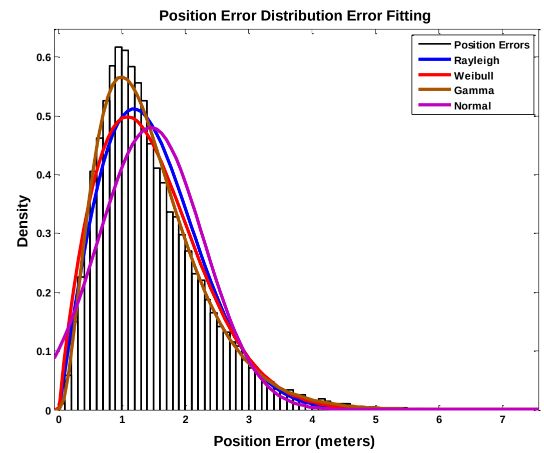

Figure 16 - Position Error Distribution

Table 1 lists the best fit parameters for these position error data distributions.

Distribution	1st parameter	2nd parameter
Rayleigh	1.18535	N/A
Weibull	1.64471	1.85666
Gamma	3.14733	0.462432
Normal	1.45543	0.831808

Table 1 - Position Error Distribution Parameters

Rather than deriving the multipliers using an analytic distribution [9], I'll use the PAF data provided by the GPSOC to derive the multipliers empirically. For each of the days analyzed, I'll calculate the navigation error at 1 minute time intervals, then sort the position, horizontal, vertical and time accuracy data and find the 50th and 95th percentile errors.
Dividing these errors by the root-mean-square error for the day provides an estimate of the one, two and three dimensional multipliers for that site for the day. To get an accurate picture of the global distribution of the multipliers, I'll repeat this analysis over the globe using a 5 degree grid. The values for each grid site are then averaged on a daily basis. The results of this analysis are plotted in Figure 17. It's interesting to see that over the 600+ days of analysis, there appear to be no trending behaviors in the multiplier data, though daily variations are quite apparent. Tables 2 and 3 compare the empirically derived multiplier values by dimension and

confidence percentage to the theoretical (Gaussian-based) values.

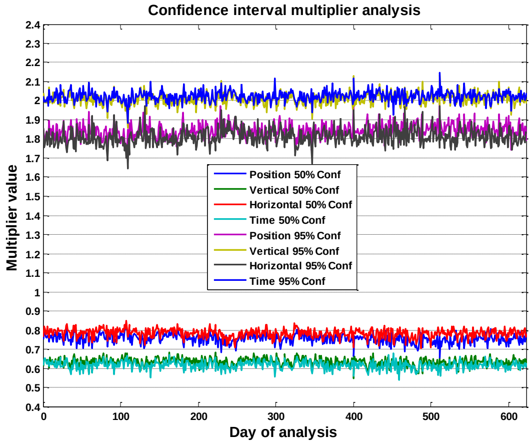

Figure 17 - Empirical Confidence Interval Multipliers

Dimensions	Empirical Value / Standard Deviation	Theoretical Value
1 – Vertical	**0.6323/0.0223**	**0.6745**
1 – Time	**0.6084/0.0220**	**0.6745**
2 – Horizontal	**0.7824/0.0236**	**0.8326**
3 – Position	**0.7551/0.0236**	**0.8880**

Table 2 - 50% Confidence Multiplier Values

Dimensions	Empirical Value / Standard Deviation	Theoretical Value
1 – Vertical	**2.0096/0.0316**	**1.960**
1 – Time	**2.0230/0.0281**	**1.960**
2 – Horizontal	**1.8109/0.0431**	**1.731**
3 – Position	**1.8433/0.0380**	**1.614**

Table 3 - 95% Confidence Multiplier Values

The variability of the empirically derived multiplier values as seen in figure 17 suggest that on a daily basis, the confidence values will not be identically 95% and 50%, but will vary slightly. For example, see Figure 18 where I compare actual position errors to 50% and 95% confidence predicted position errors.

For this particular day, the percent of actual errors outside of the 95% confidence level is 6.8% - not the 5.0% we would expect. Similarly, for the 50% confidence level data, this particular day saw 51.8% of the actual errors above the predicted errors.
The next question is then, how long can I use a single 7-day PSF file to represent navigation errors accurately, within a given confidence level? To decide this, we must take into account the variability of the empirical multipliers we use to arrive at a given confidence level – and look for excursions beyond this inherent variability.

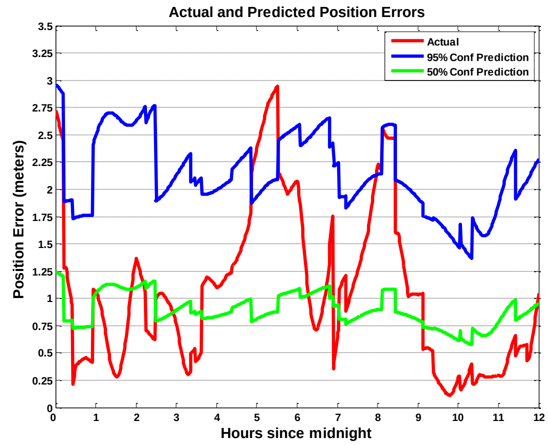

Figure 18 - Actual versus Statistically Predicted Errors

Figure 19 shows one way to visualize these excursions. A statistical prediction of GPS accuracy was made each day for 155 days past the prediction epoch. Each day, the actual navigation accuracy at a specific site was calculated, and then the predicted accuracy was calculated using the 7-day PSF file from the prediction epoch only. The 7-day PSF file was not updated as the prediction day advanced. This figure shows the percent of actual navigation errors that are greater than the 95% confidence level predicted navigation error. I'm using the term *excursions* for this quantity. This is the interesting behavior we are interested in – we want to know the actual navigation errors that are greater than our prediction and, hopefully to be able to minimize them.

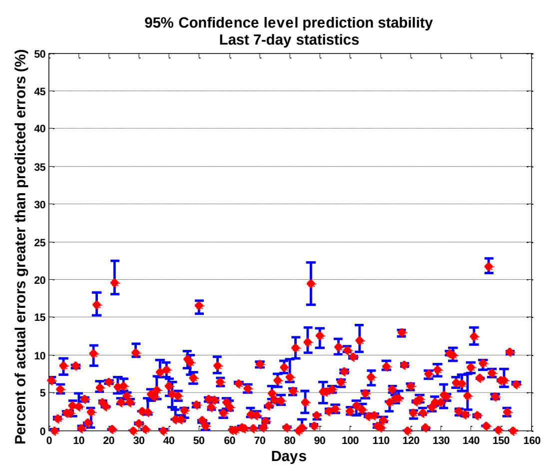

Figure 19 - Confidence level prediction stability using last 7 day statistics

Figure 19 has a few interesting points:

a) There is no apparent decrease in confidence in this graph as the prediction time increases. This would be signaled by an increasing trend in the data from left to right.

b) There is much variability in the 95% confidence predictions. At a 95% confidence level, based on the variability in the multiplier calculations of Figure 17, we'd expect a smaller variation of the actual percentage about the 5% line. Instead, we see a larger variation.

The preliminary conclusion to draw from this data is that there appears to be no time dependence on the use of the 7-day PSF file for prediction purposes. When using the 7-day PSF file for predicting navigation accuracy, and then converting to a specific confidence level, one must not expect that exact confidence level to be strictly upheld, even in the shortest prediction times. The inherent variability of GPS statistics precludes us from being able to precisely determine statistical predictions with great confidence. Standard error theory procedures do not appear to hold well when applied to GPS error measurements and further study on this topic is warranted. [9]

Figure 20 shows that 60% of the actual error excursions are within the 5% boundary. In fact, 90% of the excursions are below 10.25% (89.75% confidence). With no time-dependent behavior to rely on, using a given multiplier to predict accuracy with a certain confidence is not for the faint of heart.

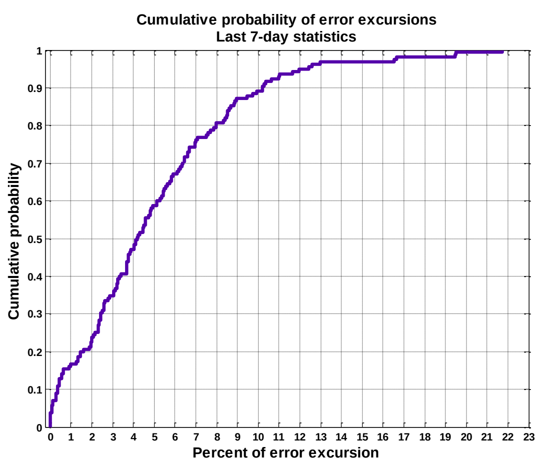

Figure 20 - Cumulative probability 95% error excursions using last 7 day error statistics

This analysis suggests that we look further into the generation of the multipliers used to satisfy our confidence interval analysis criteria. Is there a spread of the multipliers as we predict further in time? Is there a better way to derive the multipliers? We can actually find the multiplier that *will* satisfy our 95% confidence (or any confidence level for that matter) by iterating over different multipliers and counting the excursions for each. To do this analysis, I iterated over multiplier values from 0 to 2.5, for each day in my 155 day sample (Jan 1, 2007 to Jun 4, 2007). The results are plotted in figure 21.

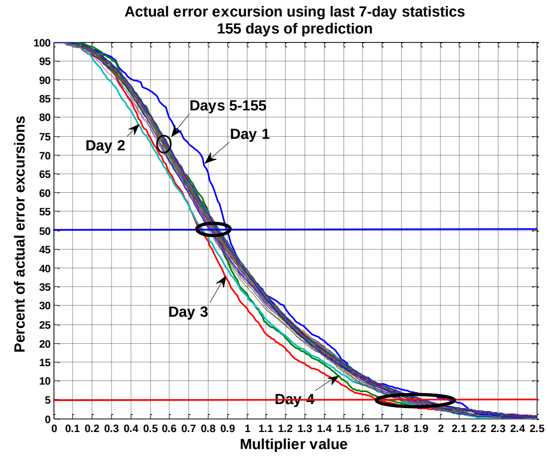

Figure 21 – Multiplier analysis for actual error excursions using last 7 day error statistics

This figure shows a familiar looking curve [9], with several lines. I've highlighted the lines for the multipliers for the number of days averaged into the prediction. This graph shows that after the first few days of prediction, the multipliers settle down to a fairly small range. The horizontal blue line signifies the 50% excursion criteria (50% confidence) and the horizontal red line signifies the 5% excursion criteria (95% confidence) I've also highlighted the width of the multiplier values for these two confidence levels. This shows that the multiplier values are:

1) Different for this regime than those obtained using the empirical method above
2) Have a fairly large spread for 95% confidence.

The shape of the curve shows us why the 95% spread is so large – the multiplier lines are almost tangential to the 5% line. Reference [12] has a good explanation for this. The multipliers have a spread of 0.75 to 0.90 for the 50% confidence level and 1.68 to 2.08 for the 95% confidence level. It appears that averaging the global multipliers derived empirically then using that single mean multiplier value may not be the best method to use.

Maximum Error Data (option 3)

The maximum errors derived in the Signal-In-Space section above could also be used to create statistical predictions of navigation accuracy. Instead of the global URE derived from the last 7 days of data for each satellite, I'll now use the maximum global URE data as the 1-sigma error in the PSF prediction scheme. To create the maximum global URE, I'll use the global URE equation (Equation 1), and use the maximum radial, along-track, cross-track and clock error statistics. Then, proceeding as above, I'll predict 155 days out and determine the 95% excursions as a function of the multiplier value required to meet that criteria. Figure 22 shows a plot identical to Figure 21, but using the maximum error statistics instead. Notice the difference in the width of the spread for the 95% confidence multipliers (along the red 5% line). This spread is much less than with the 7-day PSF file prediction. These multipliers have a spread of 0.55 to 0.68 for the 50% confidence level and 1.2 to 1.43 for the 95% confidence level.

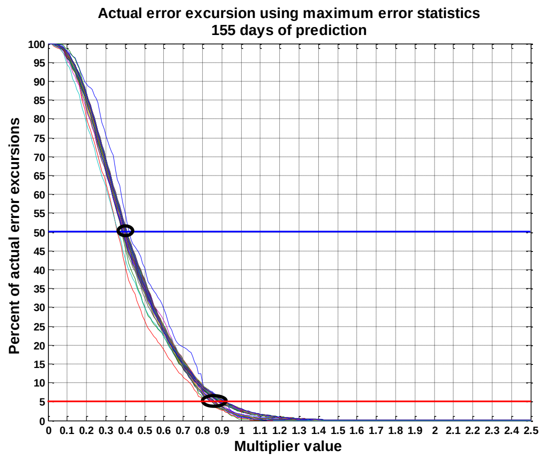

Figure 22 - Multiplier analysis for actual error excursions using maximum error statistics

To see how the excursions behaved, Figure 23 was created. This figure looks quite similar to Figure 19, in fact it's difficult to glean any new information by studying these two graphs alone.

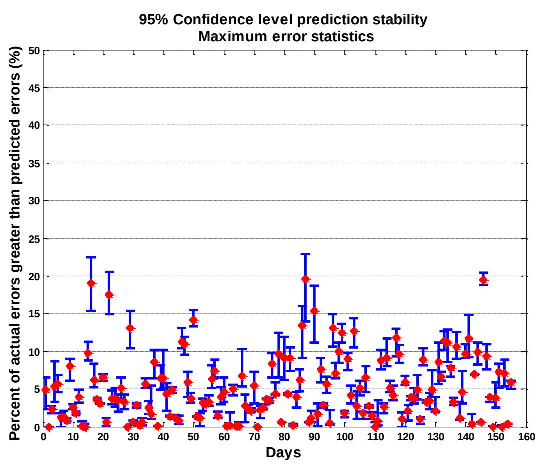

Figure 23 - Confidence level prediction stability using maximum error statistics

Looking now to see if this new maximum error approach is any better or worse, I created Figure 24, the cumulative probability plot, similar to Figure 20. With this new prediction scheme, I still have 60% of my errors within the 95% confidence level, and 90% of my errors are now within 89% confidence. The results here are not statistically significant.

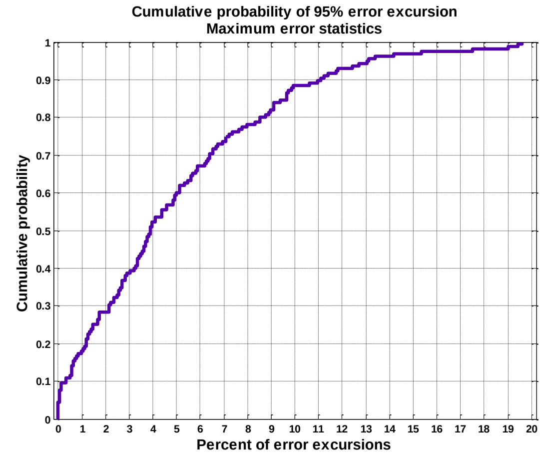

Figure 24 - Cumulative probability of actual error excursions using maximum error statistics

One final piece of analysis we can perform on these two types of predictions data is to scatter plot them. We'll look for any deviations that may show us one type of prediction method is better than another.
In Figure 25, the black diagonal line is the y = x line, the 7-Day error excursions are plotted along the x axis and the maximum error excursions are plotted along the y axis. Each blue dot represents the error excursions for one day of prediction, with the first 14 days of predictions highlighted in orange. I determined a least squares fit to the 155 days of data and found the slope of the best-fit line equal to one. Essentially, the y = x line *is* the best fit line in the graph. Thus, there appears to be no significant difference between these prediction methods when used to try to reduce the number of excursions beyond the 95% confidence level.

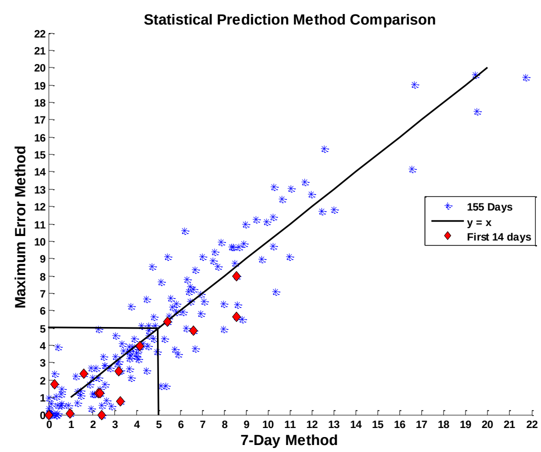

Figure 25 - Maximum Error vs. 7-Day prediction methods

We know from the previous section on DOP prediction, that we can successfully predict DOP two weeks with an almanac. With this in mind, I highlighted the first 14 days of statistical prediction in Figure 25, wondering if there was some pattern that these excursions took to lie about the best fit line. As is apparent from the graph, the pattern of the first 14 days is not different from that of the whole 155 day dataset. In fact, though not plotted, each successive month of predictions was analyzed, and found to have this same general behavior, leading to a final conclusion – there is no time dependence to statistical error excursions when predicting GPS accuracy.

Time Regimes
Now that we've explored the prediction behaviors in different data regimes, how can we make use of this information when we need to predict in a particular time regime? Should I use PAF data and extrapolate to get my navigation errors at some future time? Should I use the statistical prediction method? The following are my recommendations based on this analysis. My recommendations are presented with the understanding that as further analysis is completed, these prediction recommendations may change.

If one has access to PAF type data, it's best to use that as far as possible. This is because the PAF type data will allow you to predict instantaneous, signed errors for times in the future, providing a specific error vector. The data analyzed above shows that PAF based extrapolations can be used for roughly 6 hours with a meter or two of error. After six hours the navigation errors begin to grow and may no longer be acceptable. The choice of how long you use the PAF extrapolation technique is directly related to how much error you can stand. In this type of prediction, I would denote *long-term* as 6 hours.

If only statistical data is available, use that as a second choice. While statistical error predictions can be made for any time in the future, the nature of the 1-Sigma prediction technique does not allow for signed errors. Thus only an error ellipsoid can be generated from this type of data, instead of an error vector.
For the statistical predictions, I would use these for what ever time span you have. Since there appears to be no time-dependence on the length of prediction time with this type of data, I can recommend its use for at least several months in advance. The tricky part of this prediction is using the correct multiplier to achieve the level of confidence you want. Using Figures 21 and 22 as guides, select the multiplier value appropriate for your desired confidence level and type of PSF data, then apply to your predicted error values. For this type of navigation, I'll define *long-term* as 5+ months.

It's apparent from Figures 21 and 22, that the larger the multiplier value used will result in fewer excursions above my desired confidence level. We could use a multiplier of 10.0 say to make all the excursions in figure 23 lie within the 95% confidence level. The problem then is that my predicted errors are so large that I don't really have insight into my problem. Judgment is required here and hopefully the analysis presented here will allow the user to make better informed decisions.

COMPLETING THE PREDICTION PICTURE

Most of the analysis in the paper has focused on Dilution of Precision and Signal-In-Space errors and their prediction, either extrapolated or statistical. These errors are always present in the GPS error budget and warrant the type of analysis seen in this paper. The other errors in the GPS error budget are also deserving of analysis and must be included to complete the prediction picture [9]. I have purposely not analyzed atmospheric errors and have only touched on how the multipath and receiver errors can be modeled. Standard error propagation models [7] can be used to add differing error sources into a single combined error prediction statistic. These methods though can only be used to provide statistical error predictions.

SUMMARY

In this paper, I have analyzed the techniques necessary to predict navigation errors using data available to the typical GPS user. I've shown that almanacs can be used to predict dilution of precision values for two weeks with little difference in PDOP values. I then went on to show how the signal in space user range error values are clamped by the fact that the 2nd Space Operations Squadron uploads the GPS satellites on a regular basis. Following that I investigated an extrapolation technique useful when predicting for up to six hours in the future. Statistical prediction techniques were then addressed, first by considering a 7-day statistical strategy then by using the maximum error method. Both methods are very sensitive to the multiplier values needed to assess the predicted errors at a specific confidence level.

This analysis was performed and is applicable to only those errors that seen on a routine basis in the GPS system. The techniques discussed here will not hold when there are clock jumps, or other perturbing forces that cause the navigation errors to be significantly larger than normal. For a clearer understanding of these modes, see [10].

Several topics have been raised that are good topics for follow on papers, including:

- Determining the correct distribution for the GPS errors and using that to derive theoretical multipliers.
- Develop a standard theory for GPS measurements, along the lines of [7]
- The curve presented in figures 21 and 22 look like normal distributions – how do these arise?
- Can PAF-based URE extrapolations be clamped by the maximum errors derived here? What do the 1-12 hour predictions look like then?

- Develop multipath and atmospheric error prediction models and fit them into this prediction strategy.

ACKNOWLEDGMENTS

Thanks to GPS Operations Center for providing the data for analysis.

REFERENCES

[1] Parkinson, Bradford W.; Global Positioning System: Theory and Applications, Volume 1, Chapter 11

[2] Misra, Enge, Global Positioning System Signals, Measurements and Performance, Chapter 5

[3] Kaplan, Hegarty, Eds., Understanding GPS Principles and Applications 2nd edition, Chapter 7

[4] IS-GPS-200D, section 20.3.3.5.1.2, Almanac Data

[5] Note that the averages are over a smaller number of days for the following satellites: PRN 12:163 days, PRN 15:550 days, PRN 17:500 days, PRN 31:237 days

[6] Chatfield, Chris; The Analysis of Time Series, An Introduction 6th edition, Chapter 5

[7] Principles of Error Theory and Cartographic Applications ACIC Technical Report No. 96, February 1962

[8] Rayleigh (σ) = Weibull $(2, \sqrt{2}\sigma)$

[9] An excellent topic for a follow on paper.

[10] GPS Integrity Failure Modes and Effects Analysis: K. Van Dyke, J. Kraemer, DOT/Volpe Center; K. Kovach, ARINC; J. Lavrakas, Overlook Systems Technologies, Inc.; J.P. Fernow, MITRE CAASD; J. Reese, GPS JPO; N. Attallah, B. Baevitz, SAIC

[11] http://www.envinav.com/

[12] http://www.gpsworld.com/gpsworld/Feature/GNSS-Accuracy-Lies-Damn-Lies-and-Statistics/ArticleStandard/Article/detail/395779